

Name:

Do not begin your test until instructed to do so. You will have 45 minutes to complete the test. The **only** items you are permitted to use are a pencil, eraser, and calculator. Any use of any other item will result in getting a 0 on the test. Keep your eyes on your own test. Getting answers in any form from others will result in getting a 0 on the test. Each problem is worth 4 to 6 points: half for showing your work (when appropriate), and half for the right answer. Box your answers. Talking while any test is out will result in a 10 point penalty.

Good luck! (Even though this is a matter of skill, not luck.)

Page	Points	Score
2	31	
3	33	
4	36	
Total:	100	

1. (4 points) Evaluate $\lim_{x \rightarrow -3} (2x^2 + 4x + 1)$.

2. (4 points) Evaluate $\lim_{x \rightarrow \pi/2} \sin x$.

3. (4 points) Evaluate $\lim_{x \rightarrow -5} \frac{x + 5}{x^2 - 25}$.

4. (5 points) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$.

5. (5 points) Evaluate $\lim_{x \rightarrow 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$.

6. (4 points) Evaluate $\lim_{t \rightarrow 0} \frac{\sin 3t}{2t}$.

7. (5 points) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$.

8. (4 points) Sketch a function satisfying the following properties:

1. $f(0)$ is undefined,
2. $\lim_{x \rightarrow 0} f(x) = 4$,
3. $f(2) = 6$,
4. $\lim_{x \rightarrow 2} f(x) = 3$.

9. (5 points) A construction worker drops a wrench from a height of 1000 feet. Its distance function is modeled by $s(t) = -16t^2 + 1000$. How fast will the wrench be falling after 5 seconds?

10. (4 points) State the definition of the phrase “ $f(x)$ is continuous at $x = a$.”

11. (4 points) Where is $f(x) = \frac{x}{x^2 - x}$ continuous? Which of its discontinuities (if any) are removable?

12. (4 points) Evaluate $\lim_{x \rightarrow 2^+} \frac{|x - 2|}{x - 2}$.

13. (4 points) Evaluate $\lim_{x \rightarrow 0^-} \cot x$.

14. (4 points) Evaluate $\lim_{x \rightarrow 5^-} \frac{2x - 1}{5 - x}$.

15. (4 points) Find a so that $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$ is continuous on \mathbb{R} .

16. (6 points) State Squeeze Theorem.

17. (6 points) Use Squeeze Theorem to evaluate $\lim_{x \rightarrow 0} |x| \sin \frac{1}{x}$.

18. (6 points) State the Intermediate Value Theorem.

19. (6 points) Use the Intermediate Value Theorem to show $2 \cos t = 3t$ has a solution.

20. (6 points) State the ε - δ definition of $\lim_{x \rightarrow a} f(x) = L$.

21. (6 points) Use the ε - δ definition of limit to prove $\lim_{x \rightarrow -3} (2x + 5) = -1$.

22. (5 points (bonus)) The function $f(x) = 1/x$ is a rational function and so it's continuous on its domain. Furthermore, $f(2) = 1/2$ and $f(-2) = -1/2$, thus by invoking the Intermediate Value Theorem with the choice $M = 0$ (which is between the outputs $-1/2$ and $1/2$), there should be an input c between -2 and 2 for which $f(c) = 0$. However, f never outputs 0. What's wrong?